Quantizing relativistic action-at-a-distance mechanics

A. F. KRACKLAUER

A wave equation for charged particles whose non quantum dynamics is regulated by a relativistic action-at-a-distance formulation of dynamics is presented. Resolutions are given for what have heretofore been the two principle obstacles to construction of such an equation. One, a quantization procedure is developed on the foundation afforded by regarding the wave aspect of de Broglie's second solution as a manifestation of a classical electromagnetic background with a Lorentz invariant energy spectral density. Two, a single parameter Lagrangian is proposed which leads to equations of motion free of certain difficulties endemic to the Fokker formulation of relativistic action-at-a-distance dynamics. Finally, comments are made regarding the possible physical and philosophical impact of these resolutions and the resulting wave equation.

—University of Houston; 1973

Contents

Introduction 2
The problem as Stated in the Literature 2
The Goal: A Manifestly Covariant Wave Equation Incorporating Interaction Between Systems 2
An overview of this work 3

I. Quantum Theory 3
A. A Survey of Attempts to Reformulate Quantum Theory 3
1. Paradoxy: The source of discord and the Motivation for Reformulations of Quantum Theory 3
2. Field-Theoretic Reformulations, A survey and Criticism 3
3. Hydrodynamic Models: Survey and Criticisms 5
4. Stochastic Probabilistic Models: Survey and Criticism 5
5. Criticisms of von Neumann, Bell, Gilson et al. 6
B. Nelson's Stochastic Process 7
1. Nelson's counterexample to proofs that Stochastic and Quantum Mechanics are Irreconcilable 7
2. A Simple Version of the Proof of Irreconcilability of Quantum and Stochastic Mechanics 7
3. Hidden Determinism in Nelson's Counterexample 7
C. Duality and Waves 8
1. A Proposal: The Background Gives Imaginable Content to de Broglie's 'Second-solution' 8
2. The Fundamental Ansatz: Energetic Equilibrium with the Background 9
3. Point Particles Exhibit the Mass Renormalization Divergence of Quantum Theory 11
4. Spin: A Manifestation of Background Polarization 11
5. Additional Fields Introduce the Canonical Momenta 11
6. Fermions are Charges in Equilibrium with A Polarized Background 12
D. The Imaginable Content of the Schrödinger Equation 12
1. Schrödinger's Equation: Liouville Equation plus Background 12
2. Wave Decompositions Give Unique Operator Equivalents for Classical Expressions 13
3. Decomposition: Resolution of Interpretational Issues 13
Appendix to Subsection D 14

II. Relativistic Mechanics 14
A. No-interaction Theorems 14
1. The Role of 'Time' in No-interaction Theorems 14
2. The Special Role of Time in CJS Theorems 15
3. The Droz-Vincent No-interaction theorem: An exploration 16
B. Relativistic Action-at-a-distance Mechanics 16
1. Fokker's Unified Lagrangian 16
2. A Unified Lagrangian without Mathematical Lacunae 17
C. Interpretational and Self-consistency Issues 18
1. Magnetic Monopoles: an Inherent Contradiction 18
2. Radiation Reaction: An Emergent, Multi-body Effect 18

III. A Relativistic Wave Equation with Interaction 19
A. An Equation for Bosons 19
B. A Fermion Equation 19
C. Conclusions 20
1. Duality and Wave-packet Collapse are Eliminated 20
2. Single Parameter Relativistic Mechanics Precludes Predeterminism 21
3. Benchmark and Guidepost 21

References 21

Acknowledgment 22

Introduction

The problem as Stated in the Literature

The following statements gleaned from the literature express clearly and effectively the nature of our problem in which many physicists have become interested.

> "...the confluence of Relativity and Quantum Theory, having been frequently cited, has not been sighted." —Kerner (1972)
>
> "...there is no existent theory of the interaction of relativistic systems.
>
> There are two major attempts at the construction of a theory of relativistically interacting systems, field theory and S-matrix theory. Neither of these theories is completely successful. In neither theory has it been possible to show either the existence of a set of self consistent axioms which define the theory or to demonstrate the existence of the uniqueness of solutions to the equations that are used in the theory. —Halpern (1968)
>
> The formulation of an internally consistent, relativistic, quantum mechanical description of interaction particle still eludes us and represents on of the great unsolved problems of modern theoretical physics. At the classical level, the action-at-a-distance formulation is, in the author's view, the best we have; its chief drawback is that, so far, no one has any idea of how to construct a quantum version of the theory. — Anderson (1967)

The Goal: A Manifestly Covariant Wave Equation Incorporating Interaction Between Systems

This work is an exploratory attempt to achieve the confluence of relativity and quantum mechanics through investigation of formulations of relativistic mechanics other than field theory and S-matrix theory. It is felt that as the presently fashionable views and understandings of relativity and quantum theory defy unification, that perhaps a less well exploited approach to either of both theories would be more amenable to unification. Herein, the alternatives are investigated with an eye to finding a combination which accommodates interaction and meets the criterion discussed below.

Before initiating such a study, it is worthwhile to establish or define a specific goal that can be recognized as a significant benchmark for an investigation of the foundations of relativistic quantum theory. In this regard, a cue is taken from nonrelativistic quantum theory, where the Schrödinger equation plays an absolutely critical role, in that it stands even if the rest of quantum theory is disregarded; however, most of the rest of the theory is vacuous without the conclusions flowing from the Schrödinger equation. Thus, the chosen goal for this work is a relativistic wave equation for an interacting system; that is, the is an equation whose solutions can be used to calculate pertinent observables for a pair or more of particles whose non quantum mechanics is governed by a relativistic formalism. The crucial criterion for such an equation, is that the equation must be Lorentz invariant, or herein simply stated: covariant. (Hanus and Janyszek, 1971) In fact, it would be auspicious if the equation were manifestly covariant.

Hence, the chosen benchmark or goal of this work is a manifestly covariant wave equation incorporating interaction between particles.

By way of contrast with existing wave equations, the Dirac equation, although covariant, is not manifestly covariant, (Sakurai, 1967, p. 97) and neither it nor the Klein-Gordon equation can be extended to include interacting systems covariantly, even though they adequately serve to describe the interaction of a particle with a field. It will also be recalled that the Breit equation, which is an extention of the Pauli equation (the Pauli equation is the second order version of Dirac's equation) to interacting systems is not covariant because of "instantaneous" interaction. In fact, to date, the principle criticism of relativistic quantum theory is that interactions are always treated in an approximate or instantaneous fashion. (Hanus and Janyszek, 1971) A search for the source of this difficulty, or reason that de delayed interaction has not been incorporated, quickly leads one to the conviction that the privileged role of time in the usual quantum formalism is the crucial obstacle. Further investigation leads to the additional conviction that this feature of quantum theory could be altered only if a radical new understanding of quantum theory itself is found.

At the same time, non quantum relativistic mechanics is perplexed on both of its fronts. (Rohrlich, 1965, p. 194) The orthodox and popular field theory approach, inspired by Maxwell, is beset by the inelegancy of not admitting a consistent variational approach; that is, the equations of motion for coupled particles and their attendant fields can not be consistently derived from a single Lagrangian. Also, field theory suffers from well known divergencies. The less well known form of relativistic mechanics, action-at-a-distance formulation inspired by Newton and relativized by Fokker (1929), although on a formal level apparently well constructed, is vexed by the ontologically noxious presence of advanced as well as retarded interaction and the mathematically noxious feature of equations which are not integrable with any local stepwise procedure. These problems are discussed at greater length in Section III. In summary, non quantum relativistic mechanics is not yet disposed to quantization.

Moreover, there are several "no-go" theorems pertaining to relativistic mechanics, that prove that relativistic Hamiltonian formulations of mechanics (of exactly the sort most likely to be quantizable) do not admit interaction between particles. These theorems shall be analyzed, and it will be shown that they depend on elements in their hypotheses intended to make them compatible with the special status that quantum mechanics gives time, which, if relinquished, nullifies their conclusion.

As these various difficulties are examined in turn, it becomes apparent that those of field theory are intrinsic and irreconcilable, while those of action-at-a-distance mechanics result only from Fokker's choice of Lagrangian. In short, it may be said that two obstacles militate against the unification

of quantum theory and relativity: one, the privileged role of time in the quantum formalism, and two, the lack of a suitable Lagrangian for relativistic action-at-a-distance mechanics. Accordingly the intermediate goals of this work are first, to uncover a formalism of quantum theory which does not put time in an exalted position, and second, to propose a suitable Lagrangian for relativistic mechanics of interaction systems.

An overview of this work

Section I, Subsection A, is a survey of three other types of attempts to reformulate quantum theory, searching for clues to a fruitful approach in their successes and failures. Subsection B resolves the paradox resulting from the existence of a stochastic derivation of Schrödinger's equation coexisting with proofs that such is impossible. In Subsection C, a physical interpretation of de Broglie waves is presented which, in Subsection D leads naturally to a quantization procedure shown to give Schrödinger's equation.

Section II, Subsection A, contains an analysis of no-interaction theorems and shows that they are the result of attempts to give time the very same privileged role that it has in nonrelativistic mechanics and quantum theory. In Subsection B, a Lagrangian is proposed which circumvents the difficulties ensuing from Fokker's Lagrangian. In Subsection C two issues bearing on the epistemological foundation of action-at-a-distance mechanics are discussed, namely, the possible existence of magnetic monopoles and advanced interaction.

In Section III the quantization techniques developed in Section I are applied to the relativistic mechanics developed in Section II to achieve the benchmark set out in this introduction; i.e., a manifestly covariant wave equation incorporating interaction.

I. QUANTUM THEORY

A. A Survey of Attempts to Reformulate Quantum Theory

1. Paradoxy: The source of discord and the Motivation for Reformulations of Quantum Theory

Even when the problems in quantum theory presented by the special role of time are disregarded, the history of quantum theory is permeated with distress, schism and heterodoxy. The debate between Einstein and Bohr being a widely known case in point. The difference of opinion can be attributed to two causes: One, the nature of the foundations of the theory, which are abstract axioms having little intuitive relations to perceptual experience; and two, paradoxes in interpretation resulting from the postulated collapse of the wave packet at the instant of measurement. With regard to the second issue, Wigner's illustration (d'Espagnat, 1971, p. 419) of this paradox is probably the most incisive. He asks one to consider a wave packet for a friend who has performed a binary measurement but has yet to report the result. Such a wave packet must contain a superposition of both outcomes and "collapse" to the value measured by the friend when he gives the report; i.e., when the friend is "measured." It is, however, a common experience that those who perform binary measurements do not exist in an ambiguous state until reporting results. At the same time, a wave packet can not be regarded a representing only the observer's state of knowledge since it must mediate in certain interactions; i.e., beams of particle diffract, therefore wave functions must have an existence different from that of ordinary probability densities.

With regard to the latter cause, the history of quantum theory suffers no dearth of attempts to reformulate the foundations on the basis of axioms that are both more intuitive and revealing with respect to interpretation. In this Subsection, an historical survey is made of the epistemological motivation for some of these attempts at reformulation; the survey is made for the sake of cultural edification and stimulation. Herein, the consequent mathematical development for an idea is not examined, but attention is focused only the conceptual framework. Purely formal or mathematical reformulations will not be considered. In any survey effort of this sort, it is necessary to choose a vantage from which to evaluate and criticize. Herein, classical physics is chosen as the basis of evaluation, as the writer believes that the failure to understand certain phenomena from within classical physics thus far does not constitute a demonstration of its inadequacy to explain these same phenomena.

Most attempts at reformulating quantum theory are motivated by a similarity between some classical and quantum phenomena; therefore, these attempts may be categorized by the source of the classical inspiration; herein three types are considered: 1) field theoretic, 2) hydrodynamic and 3) stochastic–probabilistic models.

2. Field-Theoretic Reformulations, A survey and Criticism

Many field theoretic reformulations of quantum theory focus on the wave nature of matter and attempt to reconcile this wave nature by associating electromagnetic type fields with particles by a mechanism other than charge. Other field-theoretic models attempt to capture the essence of quantum theory in the mathematical character of field theories. (Jammer, 1966) In any case the basic motivation of these attempts seems to come from an intuitive idea that the success of electrodynamics must be indicative of the fundamental nature of of physics theories. The physics of these attempts is strictly of an apologetic nature, that is, these theories are justified in terms of being able to yield results comparable to laboratory experience rather that being theories deduced from fundamental intuitive concepts of innate physical appeal.

Many field-theoretic models were given a conceptual genesis by de Broglie (1922) and 1923, where he posited a reconciliation of the wave properties of light and the photoelectric and Compton effects. De Broglie demanded that the Planck hypothesis be consistent with electromagnetism via relativity by setting:

$$h\nu = mc^2. \tag{1}$$

In 1927 de Broglie successfully defended this notion in his

dissertation; however, the interpretation continued to disturb him. To wit:

> I saw not less clearly what considerable difficulties one was going to encounter when one would wish to state precisely the exact relationship of wave and corpuscle and to reconcile the two expressions of this dualism, without deviating too much from the classical ideas admitting the possibility of representing all physical realities by pictures in the framework of space and time, and attributing a vigorous determinism to their evolution. (de Broglie, 1955)

In the years following, he attempted to resolve these problems through his "theory of the double solution;" a theory which maintained that an ordinary wave function, whose phase contained the wave-like aspects of particle trajectories, and whose amplitude only reflects the probability of presence in any given region of space, was only the shadow, as it were, of a "second solution" to another wave equation. This second wave equation would have solution singularities in the wave amplitude, which should exhibit the properties of corpuscles. Thus, the singularities should ride along on the phase waves, and the whole should jibe with both the classical conception of space-time and with the observed corpuscle nature. A version of this theory with simplified mathematics was presented to the Solvay Congress in 1927, where it encountered severe objections; thereafter, de Broglie became an apostle of the orthodox view, overwhelmed by the might of opinion and by his own failure to develop this idea further. (de Broglie, 1962)

This particular line of heterodoxy, initiated by de Broglie, was revived by Bohm (1952) and (1952a) in which, in effect, he disproved by counterexample the implicit claim made by the Copenhagen School, that their's was the only internally consistent interpretation of quantum theory possible. The result was obtained, essentially, by noting that if $\Psi(x,t)$ is written as

$$\Psi(x,t) = R(x,t)e^{\frac{iS(x,t)}{\hbar}}, \tag{2}$$

and put into the Schrödinger equation, the following two coupled equations are obtained:

$$\frac{\partial R(x,t)}{\partial t} = \frac{-1}{2m}\left(R(x,t)\nabla^2 S(x,t) + 2\nabla R(x,t)\cdot\nabla S(x,t)\right), \tag{3}$$

and

$$\frac{\partial S(x,t)}{\partial t} = \left(\frac{\nabla^2 S(x,t)}{2m} + V(x) - \frac{\hbar^2\nabla^2 R(x,t)}{2mR(x,t)}\right). \tag{4}$$

These equations lend themselves to direct interpretation in the limit $\hbar \to 0$; Eq. (4) becomes the usual Jacoby equation for an action function, $S(x,t)$, whose gradient is, as in classical mechanics, a velocity field; Eq. (3) becomes the continuity equation for $\rho(x,t)$ when it is defined as

$$\rho(x,t) \equiv R^2(x,t), \tag{5}$$

and can be interpreted as a density on phase space. When $\hbar \neq 0$, however, the extra term in Eq. (4) is said to be a "quantum potential," which, in effect, introduces the "pilot wave" effect sought by de Broglie twenty years earlier. However, Bohm attached physical, rather than purely formal, significance to the waves associated with the particle through the function $S(x,t)$; thus, the collapse of the wave packet paradox (a.k.a. non-locality) must also plague this interpretation. This analysis has also been extended to the relativistic case; and, in all, the program can be held to be successful in that it is not in conflict with observations, although it provides no testable conflict with the orthodox interpretation (Freistadt, 1957), nor has it been successful at establishing a basis for quantum theory more compatible with intuition.

A more intuitively appealing concept, on the surface, is offered by Goedecke (1964), who suggested that the discrete levels of a quantized system may correspond to motions of a charged particle, which, due to geometric considerations, do not radiate. This notion was pursued further by Grimes who argued that certain other effects thought to be quantum mechanical in nature might be understood classically. However, because of the incomplete and largely programmatic nature of this concept it is open to much criticism.

Sachs and Schwebel (1961) have proposed a much grander version of a field theoretic reformulation of quantum theory. The foundation of their theory is the notion that the object of primary interest is the interaction rather than the subjects of these interactions. (Sachs, 1963) This theory, containing coupled nonlinear field equations equal in number to the number of particles in the system, interprets quantum mechanics as a statistical theory of elementary interaction. Many of the results of orthodox quantum theory can be duplicated in the reformulation, although, as the authors point out, precise agreement is in principle impossible. (Sachs, 1971) Furthermore, the source of the randomness necessitating the statistical approach and the value of $\hbar$ are also *ad hoc* addenda. The meta logic seems to be that of curve fitting; i.e., data can be fitted to a theory to within arbitrary precision by using ever increasing complexity the way a curve can be fitted by using ever more terms.

The last example herein of what can be called a field theoretic foundation for quantum theory is Wesley (1961), whose interpretation, although similar to the de Broglie-Bohm pilot wave theory, differs from it in that he considers wave functions to have no direct physical significance. Their only purpose is that of a generating function for the mathematical expressions for the orbits. The wave-like phenomena of quantum theory, according to this view, are merely a manifestation of the collective phenomena of individual particles, this has the desirability feature that, if true, it should lead to an observable discrepancy at some level between collective wave-like behavior and individual trajectories. However, to quote this author,

> ... [it] suggests that submicroscopic phenomena may be amenable to investigation with the fruitful tools of classical physics, [but] it still fails to present the actual physical problem being solved.

3. Hydrodynamic Models: Survey and Criticisms

Hydrodynamic reformulations are in fact mostly reinterpretations resulting from a coincidence in the form of certain equations. These models as a group are the least appealing, the physics is entirely apologetic, and philosophical issues are obfuscated and exacerbated.

Madlung (1926) seems to have originated the hydrodynamical model of quantum theory. In a manner parallel to Bohm's, he separated the Schrödinger equation into the real Eq. (3) and the complex Eq. (4) and noted a similarity with the Navier-Stokes equations; this model has been further discussed and enlarged by Takabayasi, Bohm, Vigier and Schoenberg; see: (Freistadt, 1957).

Two immediate objections can be raised. One, since Schrödinger's equation involves configuration space rather than physical space variables, either the model requires a separate fluid for each particle, or, the proposed fluid flow is purely a sematic simile. Two, what in the Bohm theory was interpreted as a "quantum potential" is in hydrodynamical models called a "quantum pressure." Quantum pressure has a nonlocal character in that it depends not only on the value of the fluid pressure (i.e., in hydrodynamical models $\Psi^*(x,t)\Psi(x,t)$ is interpreted as a pressure) at a given point, but also on its derivative, which necessarily involves values of the fluid pressure at neighboring points. (Bohm, 1952) This second objection has to some extent been met by Gilson (1969), who has shown that the form of a "quantum pressure" is reasonable on classical grounds based on a thermodynamic argument assuming that two fluids are in thermodynamic equilibrium.

Various further conjectures have been made by Bohm, and others, that particles with spin can be modeled or understood by considerations involving vorticity in the "quantum fluid;" he also argued that quantum dispersion can be understood as random fluctuations in these fluids. Although these two notions have a certain innate appeal, they don't overcome the repulsiveness of multiple fluids; although suggestions that only for each *type* of particle is there a separate fluid, do ameliorate this objection somewhat.

4. Stochastic Probabilistic Models: Survey and Criticism

Stochastic-probabilistic reformulations of quantum theory are both the most successful and the least apologetic. there are two principle sources from which they derive their inspiration: stochastic mechanics and some version of mathematical monism. Although early attempts to deduce quantum theory predate Chandrasekhar (1943), his paper has become the mother lode for many later attempts. The physical motivation and analysis in these attempts parallels that for Brownian Motion, and in fact often equates them.

Mathematical monism is a philosophical position holding that all physics is contained in some area of mathematics. Many relativists hold, for example, that physics is contained in geometry. (Graves, 1971) In the case at hand, probability is credited with physical content.

By far the most popular reasoning in these models is no more that algebraic machinations described with the vocabulary of statistical mechanics. In fact, although many of the these theories contain calculations free of error, deeper analysis reveals that their content must be null in order to be consistent with their interpretation; that is to say, it has been shown that quantum mechanics and stochastic mechanics are fundamentally disjoint.

Stochastic models can be categorized according to the source of the random input implicit in the term stochastic; some authors attribute the random element to an internal or intrinsic effect, such as observational error for example, while others invoke an external source. Some stochastic theories are of a purely formal sort, not imploring any physical motivation or justification, rather just exploiting analogy with equations from stochastic mechanics in order to ferret out properties of solutions with greater ease.

The mainstream of the stochastic reformulations of quantum theory begins with studies done by Wigner (1932) in a very orthodox spirit. He introduced, in connection with questions regarding thermodynamic equilibrium, the following expression relating quantum wave functions to phase-space density-like expressions:

$$P(x,p,t) = \left(\frac{1}{\hbar\pi}\right)^n \int\int\cdots\int d^n y\, e^{\frac{i2y\cdot p}{\hbar}} \Psi^*(x-y,t)\Psi(x+y,t). \tag{6}$$

The temptation to consider $P(x,t)$ a probability in the classical sense must be resisted, according to Wigner, as it is easy to discover that $P(x,t)$ need not be everywhere positive; for verification one need only take the first excited state of the harmonic oscillator for $\Psi(x,t)$ in Eq. (6). The study of this correspondence was furthered by Moyal (1947) who studied quantum theory as a statistical mechanics of indeterminate processes. He makes the observation that the eigenfunctions of the dynamical equation of a Markoff process lead, even in classical probability theory, to densities which are not everywhere positive; acceptable probabilities then are linear combinations of these eigenfunctions. Furthermore, he relates a demonstration, credited to Bartlett, that a probability $P(x,t)$, which is nonnegative for any given value of t, will remain nonnegative for all values of t under evolution determined by the Schrödinger equation. Thus, Eq. (6) establishes a link with classical statistical mechanics, albeit, purely formal; however, this link is in the wrong direction, from quantum to classical, to be useful in resolving interpretational obscurities in quantum theory. Also, the seeming innocuous role played by $\hbar$, contrasts markedly with the fact that it scales *physical* effects.

Landé (1965) and (1965a) also promulgates an exposition of quantum mechanics based on probability to resolve the dilemma resulting from the concept of duality. He has advanced much stinging criticism of the Copenhagen interpretation, and has attempted to resolve the issues by introducing a "third fundamental law" of quantum theory, as well as by arguments regarding the character probabilities must have in order to function as do those in quantum theory. This "third principle," due to Duane (1923), appears to be no more than the observation that Fourier analysis leads to a countable set of coefficients when the domain is finite (compact), which speaks to the discreteness in quantum theory. Fundamental

in Landé's analysis is the assumption of reversibility, which first he justifies as corresponding to time symmetry in classical physics, and then uses to extract the superposition principle of wave functions. In all, Landé's formulation involves less intense philosophical apology, but somehow still suffers from the impression of being teleologically motivated, probably resulting from lack of physical justification for critical points at the onset. His view also suffers the deficiency of introducing a fundamental constant without a fundamental justification, a difficultly he recognizes. (Landé, 1960)

Another example of a statistical reformulation is by Liebowitz (1968), who develops the stationary Schrödinger equation from a "mechanics of finite precision." The argument is lucid and classical but also fails to justify the quantum mechanical superposition principle as applied to solutions of the Schrödinger equation. Collins (1969) and (1970) also propose a reformulation of physics including quantum theory, in which the fundamental *Ansatz* is, that uncertainty is endemic to all physical measurement. From this *Ansatz*, and explicit probability-monism, this paradigm motivates a coterie of formal manipulations to get expressions resembling well known formula; however, since uncertainty inherent in measurement can in principle be reduced infinitely, unless assumed otherwise, the relationship with quantum theory and Heisenberg uncertainty is left obscure. Furthermore, it does not explain how quantum phenomena arise independent of observation (i.e., why does, for example, an atom not "run down" even even when not being observed?).

Many stochastic models of quantum theory depend on Brownian motion for inspiration and vocabulary; but, under analysis it often goes no further. There are two lines of attack for Brownian models, the analytic and synthetic, with the former, one shows that the equations of quantum theory can be reduced to some variant of those of stochastic theory, whereas the latter reverses the procedure.

The analytic line was initiated, apparently, by Fürth (1933) and extended by Comisar (1965), de la Peña-Auerbach and Colin (1968), and others. Comisar's analysis is dependant on Feynman's path integral approach to quantum theory in that he reduces Feynman's integral equation for the wave function to Schrödinger's equation with arguments inspired by Brownian motion theory. The results of the analysis are purely formal as it involves "fictitious collisions" and an imaginary diffusion constant.

de La Peña-Auerbach et al. (1968), are much more convincing. Their analysis proceeds in a manner parallel to that of Bohm. Making the same substitution, Eq. (2), he puts the resulting expressions in the form of the continuity equation, Smoluchowski's equation and an expression of energy conservation. He also gives a self-consistent stochastic interpretation of quantum mechanics.

The analytic approach also has a long lineage starting with the work of Feyes (1952), Weizel (1954), Kershaw (1964), Nelson (1966) and de La Pnña-Auerbach (1967) and others. Kershaw and Nelson develop develop Schrödinger's equation with arguments taken directly from stochastic mechanics, differing in what appears to be only innocuous detail. However, upon close scrutiny, one finds that the argument involves a Gaussian transition function P, to be used in the Chapman-Kolmogorov equation, (Chandrasekhar, 1943):

$$\rho(x,t+\triangle t) = \int P(x,\delta x,t,\triangle t)\rho(x-\delta x,t)d^3x, \qquad (7)$$

which, as can be verified easily, will not yield the spreading wave packet of quantum theory, rather that of diffusion theory (See: Subsection B, Section I).

A second variation of the synthetic approach can be found in the work of Marshall (1963), Surdin (1971), and Santos (1972). Marshall extends (Moyal, 1947) to charged particles and shows that the finite ground state energy of an harmonic oscillator can be understood by considering a random electromagnetic field permeating space with which any charged oscillator must be in energetic equilibrium. He further shows that this hypothesis can be extended to models of the magnetic susceptibility of materials. This viewpoint is given further weight by Boyer (1969) where it was shown that the Planck spectrum and the Casimir effect follow classically from this hypothesis. With the additional assumption that the spectrum of this background is such that it does not lead to velocity dependent forces on particles subject to it, Boyer is able to clarify photon duality. That is to say, he shows that the expressions which have been derived from the photon hypothesis and verified experimentally can also be derived from non quantum electrodynamics when effects due to this background are taken into account. (Boyer, 1968) Surdin goes a step further with the same hypothesis and advances a derivation of Schrödinger's equation, credited to Olbert (without reference), (Hayakawa, 1973), in another context; however, this derivation uses the mean value theorem in a questionable way. Santos derives further results in a non rigorous fashion; de La Pnña-Auerbach (1967) has also derived Schrödinger's equation for stochastic mechanics, but at the expense of introducing a very artificial expression for the potential.

5. Criticisms of von Neumann, Bell, Gilson et al.

In the above survey of concepts for reformulation of quantum mechanics on a basis respecting intuition, many can find an attractive alternative; however, the well known results by von Neumann (1955) and Bell (1965) raise further objections and complications. Von Neumann's theorem proves, ostensibly, the incompatibility of quantum mechanics with a 'hidden-variable" theory which would reduce to quantum theory when the heretofore hidden variables are averaged out. But, von Neumann's theory has been criticized precisely where any theory is vulnerable, namely, for having a condition in its hypothesis that is not germane to every sort of hidden variable theory, namely that the hidden variable are linear in the same sense as the canonical variables. Bell's theorem shows that no local hidden variable theory can reproduce exactly quantum mechanics. It also leads to experiments which can decide between a local hidden variable theory and orthodox quantum theory; thus far (1973), results favor the latter. However. Bell's work does not exclude the possible of a classical foundation for quantum theory since it is not made clear whether

classical theories are local theories.[1]

Stochastic theories have another obstacle. Gilson (1965) has shown that: "... quantum mechanics has little if anything to do with stochastic theory."

Using an argument based on Feynman's propagator, he found that, one, a quantum transition function is dependant on the initial state; and two, that a quantum transition function is more like a delta function that a Gaussian. The same results were obtained by Hall and Collins (1971) based on a study in which stochastic mechanics was rendered in the language of operators on a Hilbert space in order to make the two theories comparable. Their conclusion that "... quantum mechanics and stochastic mechanics do not coincide except in a trivial case," follows from the observation that quantum operators are unitary (reversible) whereas those of stochastic theory are isometric (generally irreversible).

Although none of these reformulations has treated 'spin,' and others are manifestly inadequate, attention should be direct to efforts which are still potentially successful, namely those of Goedecke and Boyer. Still, even though no formal refutation of Goedecke's notion is known, it seems very unlikely that the myriad of quantum phenomena can be clarified by his idea. On the other hand, Boyer's work has had success at elucidating some the phenomena which stimulated the inception of quantum theory, including 'photon statistics,' and therfore can be without consequence.

B. Nelson's Stochastic Process

1. Nelson's counterexample to proofs that Stochastic and Quantum Mechanics are Irreconcilable

The following issue on the foundations of quantum mechanics has come to something of a paradoxical deadlock. On the one hand Nelson (1966), and others, encourage the inference that stochastic processes play some sort of ill understood role in quantum phenomena, while on the other hand, Gilson (1965) and Hall and Collins (1971) have shown that stochastic and quantum mechanics coincide only in the trivial case of deterministic stochastic processes. The point of this Subsection is twofold: first, to exhibit a simple argument which leads to the conclusion of the latter authors, and second, to resolve the apparent paradox by drawing attention to certain elements implicit in the assumptions employed by Nelson in his successful derivation of Schrödinger's equation, and to show, that in spite of the physical motivation, Nelson's formalism is compatible with the conclusions of Gilson, Hall and Collins.

[1] *After-the-fact note:* This statement was written early in 1973, when it was unclear, to this writer at least, whether Bell's and Einstein's non-locality were the same. Subsequent publications by Bell in the late 1970's and 1980's removed all ambiguity; and, this writer then took the view that Bell too must have introduced an inappropriate hypotheses. Recent work has solidly verified this hunch.

2. A Simple Version of the Proof of Irreconcilability of Quantum and Stochastic Mechanics

This argument is in the form of a counterexample to claims that quantum mechanics can be modeled by a Gaussian process, and proceeds as follows: consider a hypothetical wave function, say:

$$\Psi(x,0) = Ae^{\frac{-x^2}{2}}. \tag{8}$$

Use Feynman's propagator to calculate $\Psi(x,t)$:

$$\Psi(x,t) = \sqrt{\frac{m}{i2\pi\hbar t}} \int dx' e^{\frac{i(x-x')^2}{\hbar/2mt}} \Psi(x,0). \tag{9}$$

Now compute $\rho(x,t)$:

$$\rho(x,t) = \Psi^*(x,t)\Psi(x,t), \tag{10}$$

and compare this with $\rho(x,t)$ computed by using a transition probability for a Gaussian stochastic process as follows:

$$\rho(x,t) = \sqrt{\frac{m}{2\pi\hbar t}} \int dx' e^{\frac{(x-x')^2}{\hbar/2mt}} \rho(x,0), \tag{11}$$

where $\rho(x,0)$ is:

$$\rho(x,0) = \Psi^*(x,0)\Psi(x,0). \tag{12}$$

Eq. (10) has the general form:

$$\rho(x,t) = \sqrt{\frac{1}{1+at^2}} e^{\frac{-x^2}{1+at^2}}, \tag{13}$$

whereas Eq. (11) results in:

$$\rho(x,t) = \sqrt{\frac{1}{1+at}} e^{\frac{-x^2}{1+at}}. \tag{14}$$

The quantum expression, Eq. (13) is the familiar spreading wave packet of quantum mechanics, whereas the stochastic expression, Eq. (14), is familiar from diffusion theory. The results are not the same, and Eq. (14) essentially constitutes a counterexample to the claim that stochastic processes may be used to model quantum theory. The conclusion to be drawn from this counter example is: Gaussian stochastic processes in configuration space (an Einstein-Smoluchowski process) can not model quantum theory. Since the very same calculation can be executed in momentum space, Ornstein-Uhlenbeck processes (i.e., a Gaussian in velocity) (Chandrasekhar, 1943) are also excluded. Although this exercise suffers, as do most counterexamples, from a paucity of insight in the issue at hand, it is offered as an expedient alternative to the more complex but more revealing arguments of Gilson, Hall and Collins.

3. Hidden Determinism in Nelson's Counterexample

Nelson's formulation appears to stand in defiance of the work of Gilson, Hall and Collins. Although the latter have

shown that no stochastic process can model quantum theory, Nelson appears to have devised a stochastic process which does so. This conflict can be resolved by showing that in fact Nelson's process is deterministic, which we propose to do by showing that it very quietly departs altogether from stochastic theory. Nelson's paradigm is motivated with a discussion of Brownian motion in configuration space, so that it is written for dx:

$$dx = b\,dt + dw(t), \tag{15}$$

where dx is an incremental change in the position x of a particle, b is the mean forward (in time) velocity, and $dw(t)$ is a "Weiner" process, i.e., a random incremental change whose moments to first order are:

$$< dw >= 0; \quad < (dw)^2 >= \nu dt; \tag{16}$$

consistent with Brownian theory. But, the paradigm also needs a second process:

$$dx = b_* dt + dw_*(t), \tag{17}$$

where b_* is to be the so-called 'mean backward velocity' and $dw_*(t)$ is a Weiner process such that $dw(s)$ is independent of $dw(t)$ for $s \geq t$. Nelson did not proffer physical motivation for this second process. However, one must conclude, that either the rearward process is the forward process expressed in a time reflected frame, or it is not. If it is not, then either it affects half of the ensemble all of the time, or all of the ensemble half of the time. In the latter case it could be expressed in the time-forward frame and added to what is already there to give a normal Weiner process; and, in the former case, quantum systems would be comprised of non-interacting forward and backward partial ensembles, each of which must nevertheless be observable to conscious quantum systems (e.g., human observers), —a paradox. If this process is a forward process expressed in the time reflected frame, then the correlation between the two process can be computed and is found to be:

$$< dw\,dw_* >= -\nu dt. \tag{18}$$

So, considering the combined process, $w + w_*$, its second moment is:

$$< (dw + dw_*)^2 >= \nu dt + \nu dt + 2 < dw\,dw_* >, \tag{19}$$

which is clearly zero, i.e., deterministic! This fact reconciles Nelson's paradigm with Gilson's, Hall and Collins' "no-go" conclusion.

C. Duality and Waves

> Mais je ne suis pas sûr que dans un univers où tous les phénomènes seraient régis par un schṁa mathématiquement cohérent, mais dépourvu de contenu imagé, l'esprit humain serait pleinment satisfait. —Thom (1972)

1. A Proposal: The Background Gives Imaginable Content to de Broglie's 'Second-solution'

In recent times no one has brought deeper doubt and more disciplined criticism to what has become the orthodox understanding of the quantum mechanical concept of duality, than the originator of the concept, de Broglie. He has accomplished this critique by his advocacy of an alternative theory, designated as the "theory of the double solution", (de Broglie, 1970) which seeks to interpret the wave character of particles in a fashion respecting the integrity of space and time and therefore our classical intuition.

Such a reinterpretation in needed not in order to satisfy human vanity by respecting intuition, but in order to resolve the paradox which perplexes the interpretation of quantum theory. This paradox, apparently discussed first by Einstein (Ballentine, 1970), can be stated briefly as follows: Suppose a plane wave solution to Schrödinger Equation, representing a particle in a beam, impinges on a plane detector such that the wave front is parallel to the detector. Eventually one observes that the particle impacts the detector at a distinct location. At the very instant of impact, the entire wave must "collapse" to the point of impact, and must do so faster than the speed of light, which implies that the wave must not be considered a "physical" wave, but instead must be regarded as an "informational" device. On the other hand, beams of particle are diffracted at slits, which implies that particles are moved in their individual trajectories apparently through the mediation of the wave. The fact that the wave mediates in physical events means that the wave must be a "physical" entity. Thus, the wave must be both unphysical and physical, paradoxically.

According to the imagery of de Broglie's double solution theory, particle duality is a manifestation of the physical nature of a particle (i.e., something whose classical limit is a particle) as "a very small region of high-energy concentration as a kind of moving singularity." (de Broglie, 1970) In other words, the wave character of a particle is due to the wave in which the particle resides as a singularity.

On the other hand, wave duality can be accounted for consistently with the classical postulate that there exists fluctuating electromagnetic radiation with a Lorentz invariant energy spectral density. Lorentz invariance insures that no frame is preferred, and implies that the energy spectral density is of the form:

$$E(\omega) = \text{const.} \times \omega, \tag{20}$$

where the constant is set equal to $\hbar/2$ phenomenologically. (Boyer, 1968)

Now Theimer (1971) has shown in a beautifully simple way how this background spectrum leads to the Planck black-body spectrum without a "quantum" hypothesis. He has also derived the following expression for the fluctuations of thermal (blackbody) radiation energy density:

$$\frac{< (\delta\rho_T)^2 >}{< \rho_T >^2} = 1 + \frac{2 < \rho_B >}{< \rho_T >}, \tag{21}$$

where ρ_T and ρ_B are the energy densities of the thermal and background fields respectively, and $\delta\rho$ is the fluctuation magnitude. The significance of this expression is that the first term on the right side is characteristic of classical waves, the second term of a classical system of particles. The sum may be said to characterize a "dualistic" entity; however, Theimer's derivation shows that this dualism need not be predicated on a quantum hypothesis, but can be understood without violating the identity of classical waves. Furthermore, this demonstration is sufficient to completely preclude the need for a "quantum" or photon hypothesis, because this need arises from the requirement to explain "photon statistics," which Eq. (21) satisfies.

It is the point of this comment to propose the argument that the background hypothesis can also be used to furnish imaginable content of a less ethereal form to the basic idea advocated by de Broglie with his theory of the double solution. (Kracklauer and Collins, 1974) In particular, it is proposed herein, that the background provides the wave to which a charged particle may be said to couple to construct the composite unit of wave and singularity imagined by de Broglie.

2. The Fundamental Ansatz: Energetic Equilibrium with the Background

The fundamental Ansatz upon which the imaginable content for particle duality is built is the claim that any particle with charge structure will obtain, when considered for suitably long periods of time, energetic equilibrium with the mode of the background to which the particle's charge structure predisposes it to couple. Alternately, this may be expressed by saying the the particle "tunes" to a particular mode from the background and establishes energetic equilibrium with background signals in this mode.

As an example, consider a dipole consisting of two oppositely charged particles held apart by a spring such that the resonant frequency of the system is ω_0. The consequence of the above Ansatz is that the time average kinetic energy of the oscillator, written usually as $mA^2\omega_0^2/2$, is equal to the time average energy in the fluctuating electromagnetic background mode ω_0, namely $E(\omega_0)$; i.e.,

$$mA^2\omega_0^2/2 = E(\omega_0). \tag{22}$$

This expression is set out here as an hypothesis, with some disregard for the details because the thrust of the analysis presented herein is directed toward an understanding of 'particle duality,' and not of the structure of the particle itself. In fact, however, Abraham and Becker (1933) have shown that Eq. (22) is rigorous to first order; furthermore, Surdin (1971) has shown that the second order approximation leads to a "Lamb" type correction. These refinements, however, are not germane to the subsequent basic argument regarding particle duality and the resolution of its concomitant paradox.

It now remains only to interpret Eq. (22) in terms of observable or known quantities. To begin, observe that the energy of the oscillating dipole is indistinguishable from the rest energy of the system to an observer who perceives only a massive unchanging system; i.e., an observer unaware of the dipole interaction with the background, who would write:

$$mA^2\omega_0^2/2 = m_0c^2, \tag{23}$$

where m_0 is, as it were, a "renormalized" mass greater than the sum of the rest masses of the charged particles comprising the dipole. The difference in mass is due, of course, to the relativistic oscillation of the particles. As it was shown above, the energy spectral density which is Lorentz invariant is given by the equation such that the energy per normal mode is:

$$E(\omega_0) = \hbar\omega_0/2. \tag{24}$$

Eq. (22) can, therefore, be written:

$$m_0c^2 = \hbar\omega_0/2. \tag{25}$$

Now, it is of interest and consequence to investigate the composition of the right hand side of Eq. (25) in greater detail. Implicit in the above development is the understanding that Eq. (25) is valid as written in the rest frame of the dipole, where it is meant to express the fact that the average energy of the system equals the average energy of the mode ω_0. The question becomes, therefore, how to express the concept of energetic equilibrium in an arbitrary frame other that the rest frame of the particle.

In order to resolve this question, a means must be found of transforming the average energy of the background mode to which the particle is tuned. A problem arises in that the time average equilibrium is established with regard to the unit of time of the particle's rest frame. This unit of time is not frame independent so that what has been computed in the particle's rest frame must be recomputed with respect to the appropriate time unit in an equivalent frame. Therefore, at once, it is seen that the averages can not be computed then transformed, rather the transform must be executed first, then the averages computed.

Time average energetic equilibrium between a dipole and isotropic signals in a particular mode in the rest frame of the dipole also implies time average momentum equilibrium since the particle's momentum is zero in this frame and the time average momentum transport of isotropic radiation is also zero. If this statement is physically meaningful, it follows that it must be frame independent; therefore, it follows that time average momentum equilibrium must also hold in each frame when computed with respect to the time unit of that frame.

There now remains only one aspect to the question of how to transform the time average energy equilibrium statement to an equivalent frame and that aspect is: how are the energy and momentum of the signals of the background expressed? It is precisely with respect to this question that the energy spectrum, Eq. (5), proves most auspicious. Consider the general expressions for the energy and momentum of plane waves in free space, to wit:

$$E = \frac{1}{8\pi}\int |E_0(\omega)|^2 d^3x, \tag{26}$$

and

$$P = \frac{1}{8\pi c}\int |E_0(\omega)|^2 d^3x. \tag{27}$$

Now, by virtue of the Lorentz invariant energy spectral density, it follows that

$$\frac{1}{8\pi}\int |E_0(\omega)|^2 d^3x = \frac{1}{4}\hbar\omega, \tag{28}$$

so that for the average of the background signals, the energy may be expressed as:

$$E = \frac{1}{4}\hbar\omega; \tag{29}$$

and momentum as:

$$P = \frac{1}{4}\hbar\vec{k}. \tag{30}$$

These expression refer, of course, to average or characteristic signals. In the rest frame of a particle there are two such signals for each direction in space corresponding to $\pm\vec{k}$. Therefore the total time average of the energy for each dimension in space is in fact:

$$E = < \left(\frac{\hbar\omega_+}{4} + \frac{\hbar\omega_-}{4}\right) >, \tag{31}$$

where $<>$ denotes time average of these two signals, so that

$$E = \frac{1}{2}\hbar\omega. \tag{32}$$

On the other hand, the total time average of the momentum:

$$P = < \frac{1}{4}\hbar\vec{k} - \frac{1}{4}\hbar\vec{k} >, \tag{33}$$

is clearly zero in the rest frame of the dipole. This result is obtained because the fluctuating background signals may be said to be one-half the time represented by a plane wave moving to the left, say, and other half moving to the right, so that on the time average there is no motion.

If now, however, the $\omega_\pm$ and $\vec{k}_\pm$ are transformed to another inertial frame and then the averages are computed, the following expressions are obtained for the energy:

$$\begin{gathered} E' = < E'_+ + E'_- >, \\ E' = \tfrac{1}{4}\hbar < \gamma\Big((\omega_0 + c\beta\vec{k}) + (\omega_0 - c\beta\vec{k})\Big) >, \\ E' = \tfrac{1}{2}\hbar\gamma\omega_0; \end{gathered} \tag{34}$$

and for the momentum:

$$P' = \frac{1}{2}\hbar\gamma\beta|\vec{k}_0|, \tag{35}$$

where a factor of $1/2$ arises with regard to momentum as an expression of the fact that each sign occurs one-half of the time; i.e., the time average of two equally probable vectors is heir barycentre.

Now, by transforming the energy of the particle and equating momentum and energy parts to the corresponding parts for the background, yields:

$$\gamma m_0 c^2 = \frac{\hbar\gamma\omega_0}{2}; \tag{36}$$

and

$$\vec{p} = \frac{\hbar\vec{\beta}|k_0|}{4}. \tag{37}$$

In a nonrelativistic approximation, the energy terms expanded give:

$$m_0c^2(1+\beta^2/2) = \hbar\omega_0/2(1+\beta^2/2), \tag{38}$$

or

$$E'_v = m_0\frac{v^2}{2} = \hbar\omega_v, \tag{39}$$

where

$$\omega_v = \beta^2\omega_0/4, \tag{40}$$

so that

$$\vec{k}_v = \frac{\omega_v}{v} = \frac{\beta k_0}{4}, \tag{41}$$

is in agreement with Eq. (37) when $\gamma \to 1$. Eqs. (39) and (41) are recognized as the classical "de Broglie relations," so Eqs. (36) and (37) can be identified as their relativistic generalizations.

Physically, the implication is that to an observer in a frame translating with respect to the rest frame of a particle with charge structure, the average or effective properties of the background electromagnetic signals with which the particle is in equilibrium can be characterized as a wave described by the well known de Broglie relations. It is this fact which gives imaginable content to the basic concept of de Broglie's theory of the double solution and which is in complete accord with notions familiar from classical physics. Furthermore, since this "average wave," as it were, is in fact the composition of classical electromagnetic waves, its response to obstacles in the environment is governed by the principles of electromagnetism. As an illustration, let us consider the pedagogical paragon, a particle beam passing through double slits. The results of this experiment can be understood as follows: The particles tune to an average effective signal which in the frame of the slits' screen is described by a wave impinging on the screen whose wave vector is the "de Broglie wave vector" of the particles. The effect of the slits on this wave is, according to the principles of wave theory, to establish a diffraction pattern on the back side of the screen. This diffraction pattern represents spacial variation of the energy of the signals in the background to which the particle is tuned, a pattern which give rise to spatial gradients of energy; i.e., of forces, which tend to coax the particle into the troughs in the pattern, much as dust settles on the nodes of a vibrating drum head.[2] The resolution of the philosophical dilemma posed by Einstein, Schrödinger

[2] *After-the-fact note:* In fact particles ride antinodes of de Broglie waves, a refinement accommodated in this paradigm by the author first in the late 1990's.

and others, is an equally straightforward application of the understanding afforded by this viewpoint.

Consider, for example, the paradox first proposed by Einstein. If a free particle impinges perpendicularly on a screen punctured by a infinitesimally small hole, then, according to the principles of Quantum Theory, the wave function of the particle beam should emerge from the hole having been refracted into a spherical wave. Furthermore, if a perfectly spherical detector is centered on the hole, then an instant before the particle impacts the detector the wave function for the particle will be finite over the entire surface of the detector. However, immediately upon impact the wave function must collapse to a zero value everywhere except at the precise location of the impact. This collapse must occur faster than the speed of light, which implies that the wave function cannot be regarded as a physical entity; but on the other hand, the wave must also mediate in the refraction, and must, therefore be physical.

The resolution of this paradox afforded by the background concept is direct and simple. The particles of an ensemble are deflected in passing through the hole by the agency of the fluctuating background so that the informational character of the wave function is freed of the preternatural task to reflect the essentially statistical nature of the fluctuations as they affect the sample paths of the ensemble.

3. Point Particles Exhibit the Mass Renormalization Divergence of Quantum Theory

The argument presented above appears to be inadequate for the understanding of point charges because they have no preferred mode of interaction with electromagnetic fields. This inadequacy is as much apparent as real. It is only apparent in the following sense. Point particles may be regarded as charge structures which interact with electromagnetic fields in a multiplicity of of modes, in this case every mode. Therefore, Eq. (32) may be written:

$$m_0c^2 = \frac{1}{2}\hbar\int_0^\infty \omega f(\omega)d\omega, \tag{42}$$

where $f(\omega)$ is an admittance function such that the integration over all modes gives a convergent result which serves as an equivalent ω_e. Following from the fact that all equations regarding de Broglie relations are linear in ω, it is permissable to replace ω_0 with ω_e everywhere. In other words, the linearity of the de Broglie expressions implies that multiple interaction with the background will not lead to different results or conclusions.

The inadequacy is real, however, in that the admittance function, $f(\omega)$ has no rationalization within the context of these considerations. This fault is, however, faithful to quantum theory where precisely this problem arises in mass renormalization calculations and is resolved only through the *ad hoc* imposition of cut-offs. (Harris, 1972) With regard to this difficulty, this author finds two possible resolutions suggested by the concept of background radiation. One, the radiation reaction to accelerations caused by interactions with the background may lead to a suitable acceptance function, $f(\omega)$. Two, the background may be Lorentz invariant only to first order, while in fact being convergent. In any case, any means whatsoever that would lead to an acceptance function is adequate for the conclusions obtained regarding duality.

4. Spin: A Manifestation of Background Polarization

A fundamental aspect of electromagnetic radiation is its two state character manifested as polarization phenomena. Since the background signals with which a charged particle are in equilibrium are electromagnetic, the consequences of polarization must be included in the fundamental Ansatz employed above. This can be most effectively accomplished by elaborating the Ansatz with the stipulation that the helicity of the particle and the 'effective' de Broglie wave be the same. Symbolically, in terms of four-vectors:

$$[\sigma\cdot\vec{p},\, \Pi\gamma m_0c^2] = \hbar[\sigma\cdot\vec{\beta}\gamma|k_0|/4,\, \Pi\gamma\omega_0/2], \tag{43}$$

where σ in this equation represents a Pauli spin vector and Π is the 2×2 identity matrix, which in this context, are nothing more than the formalistic devices through which the two states of polarization are taken into account. The "contenu imagé" of this stipulation is the following: a point charged particle can tune to either of two waves, which may be thought of as clock- or counter-clock-wise polarized in an arbitrary frame. To an observer in this arbitrary frame, the particle will appear to be driven in either right or left hand helical motion of the same sense as the effective de Broglie wave to which the particle is tuned. Of course, this naive imagery is overstated and in fact unnecessary. A more realistic image would be that of an ensemble of identical particles in interaction with a randomly polarized signal of the background whose statistical properties (expectations) are identical to the ideal situation in which particle execute perfect helical motion. In any case, spin is in evidence only in the presence of a magnetic field.

This model of electron spin is by no means unique to this author. Smit (1959) in his book on ferrites uses the model to comprehend certain phenomena in these metals. By employing a novel formulation of tensor analysis, called "space-time algebra," Hestenes (1973) has shown that what is here presented as an hypothesis, is in fact, a consistent interpretation for the Dirac equation. In effect, Hestenes reversed the logic of arguments used herein, as the Dirac equation shall be extracted below.

5. Additional Fields Introduce the Canonical Momenta

Thus far in these considerations it has been tacitly assumed that the only field with which the particle is interacting is the fluctuating electromagnetic background. In the presence of additional fields the fundamental *Ansatz* must be altered to include the energy of these fields as follows:

$$m_0c^2 = \frac{1}{2}\hbar\omega + e\Phi, \tag{44}$$

where e is the charge of the particle so that $e\Phi$ is the energy of interaction with the additional field. If the extra term, $e\Phi$, is moved to the left side of Eq. (44), and the transformation to a translating frame is carried out, the following expression is obtained:

$$[\vec{P},\Pi(\gamma m_0 c^2 + e\Phi)] = \hbar\gamma\beta|k_0|/4, \Pi\hbar\gamma\omega_0/2, \tag{45}$$

where $\vec{P}$ is canonical momentum:

$$\vec{P} = mv - \frac{e}{m}A. \tag{46}$$

In the context of the present considerations this conclusion plays no vital role; however, it will be found useful in Subsection D in which the canonical momenta is fundamental.

6. Fermions are Charges in Equilibrium with A Polarized Background

There appears to be no limit to the imaginary construction that result from compounding considerations of the above sort. For example, the Pauli Exclusion Principle might be rendered as the statement that two point charges in proximity will tend to equilibrate with oppositely polarized background waves, as each particle being driven in circular motion is an effective magnetic dipole and magnetic dipoles energetically prefer to antialign. As a second example, a massive Boson can be though of as a bound combination of fermion which as a unit equilibrate with the background scalar wave composed of the sum of two polarized background waves. Other examples can also be found.

D. The Imaginable Content of the Schrödinger Equation

In the preceeding Subsection it was shown that the particular nature of the background with a Lorentz invariant spectral density engenders to first order a statistical effect which gives the appearance to a particle of having a "de Broglie wave" attached to it. In this Subsection, this concept is exploited further by decomposing the density for Gibbsian ensemble in terms of Fourier components, each of which is such a de Broglie wave.

1. Schrödinger's Equation: Liouville Equation plus Background

It was shown in Subsection C that the background leads to an effect which causes particles to behave as if they had de Broglie waves attached to them. There it was shown that this wave, with wave vector k, is associated with the particle's momentum p by:

$$2\frac{p}{\hbar} = k. \tag{47}$$

This allows for the Fourier decomposition of a Gibbsian ensemble density $\rho(x,t)$ as follows:

$$\hat{\rho}(x,x') = \int e^{\frac{i2p\cdot x'}{\hbar}}\rho(x,p)dp. \tag{48}$$

In like fashion, transforming the Liouville equation for $\rho(x,p)$, yields:

$$\frac{\partial\hat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)\left(\frac{\partial^2}{\partial r\partial r'}\right)\hat{\rho} - \frac{i}{\hbar}(x')\cdot F(x)\hat{\rho}. \tag{49}$$

To separate variables, this equation can be tranformed with:

$$r = x + x'; \quad r' = x - x', \tag{50}$$

to give:

$$\frac{\partial\hat{\rho}}{\partial t} = \left(\frac{\hbar}{i2m}\right)\left(\frac{\partial^2}{\partial r^2} - \frac{\partial^2}{(\partial r')^2}\right)\hat{\rho} - \frac{i}{\hbar}(r-r')\cdot F\left(\frac{r-r'}{2}\right)\hat{\rho}. \tag{51}$$

This equation, however, is still not separable because of the form of the term $F\left(\frac{r-r'}{2}\right)$. Nevertheless, we note that all calculations of expectation values and densities on configuration or momentum space require the solution to Eq. (51) only along the line defined by $r = r'$. The following example proves this assertion for expectations of configuration variables, the others are shown in an appendix. Consider the calculation for the expectation $< x >$:

$$< x > = \int\int dpdx x\rho(x.p), \tag{52}$$

which, with Eq. (48), becomes:

$$< x > = \int\int\int dxdx'dp\, x\, e^{\frac{-i2p\cdot x'}{\hbar}}\hat{\rho}(x,x'), \tag{53}$$

with the change of variable, Eq. (50), this becomes:

$$< x > = \int\int drdr'\,\delta(r-r')\left(\frac{r+r'}{2}\right)\hat{\rho}(r,r'). \tag{54}$$

The appearance of the Dirac delta function, $\delta(r-r')$, demonstrates that point that the solution is needed only along the line defined by $r = r'$.

The following arguments are now employed to obtain the solution along the line $r = r'$. Noting that Eq. (51) has the appearance of a separable equation when $r = r'$, write

$$\hat{\rho}(r,r') = \Psi^*(r')\Psi(r). \tag{55}$$

Putting Eq. (55) into Eq. (51) yields, with the same manipulations used to separate variables, the equation:

$$\frac{\partial\Psi}{\partial t} + \left(\frac{\hbar}{i2m}\right)\frac{\partial^2\Psi}{\partial r^2} + \left(\frac{i}{\hbar}\right)r\cdot F\left(\frac{r+r'}{2}\right)\Psi(r) = C(r,r')\Psi, \tag{56}$$

and its complex conjugate, where $C(r,r')$ plays the role of what could be called a "function of separation." This procedure is, of course, a non rigorous contrivance; however, it is proved in an appendix the Subsection that a solution of Eq. (56) with r set equal to r' leads to a density $\hat{\rho}(r,r')$ which is a solution of Eq. (51) on the line defined by $r = r'$; further more, configuration expectations calculated with $\Psi^*(r')\Psi(r)$ are identical to those calculated with the full solution to

Eq. (51) on the entire $r - r'$ plane as $< p^1 >$, if the function $C(r)$ is chosen to satisfy

$$\frac{d < p >}{dt} = - < \nabla V(x) >, \tag{57}$$

where $V(x)$ is the potential energy function which leads to the force $F(x)$, and the expectation values are computed as above.

Demanding that $C(r)$ be chosen such that Eq. (57) is valid, is not the imposition of a new stipulation, but the reintroduction of information lost via the separation technique. Eq. (57) is trivially valid whenever the densities used to compute the expectation values of quantities appearing in Eq. (57) satisfy the Liouville equation. Therefore, demanding that $C(r)$ be chosen so that Eq. (57) holds, is a means whereby of all densities of the form of Eq. (55) which coincide with solutions of Eq. (51) on the line $r = r'$, those which also yield expectation values of momentum identical to those computed with the solutions to Eq. (51), are chosen. Note that if $C(r, r')$ depends on Ψ , so does V, consistent with present practice.

Now, by calculations parallel to those used in proof of Ehernfest's theorem, Eq. (57) and be shown to imply that

$$\int dr \Psi^*(r)[r \cdot \nabla F + \nabla C]\Psi(r) = 0. \tag{58}$$

With assistance of a vector identity, $[\cdot]$ becomes:

$$[r \cdot \nabla F + \nabla C] = \nabla[r \cdot F + C] - F, \tag{59}$$

or

$$r \cdot F + C = -V. \tag{60}$$

So that finally, Eq. (56) becomes the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + v\Psi. \tag{61}$$

The question is, just what has this procedure achieved? The answer is, that two equations have been found, (Eq. (61) and its conjugate), for which the product of the solutions satisfy the non separable Eq. (56) on a restricted domain ($r = r'$). By further demanding that the separable form $\Psi^*\Psi$ yield the same result as $\hat{\rho}(r, r')$ in Eq. (57), effectively the Ψ are so constrained so as to be identical with $\hat{\rho}(r, r')$ on the line $r = r'$; so that first power expectation values calculated either way are identical. Thus separable equations have been found, for which the products of their solution products are identical with the solution to a non separable (Liouville) equation on the line $r = r'$. By good fortune, the separable solution satisfies most every calculational need.

Others, e.g., Surdin (1971), have used essentially the same argument with different motivation and analysis; however, the more revealing approach presented here resolves certain issues discussed below.

2. Wave Decompositions Give Unique Operator Equivalents for Classical Expressions

Classical quantization rules give an algorithm to construct equivalents for classical expressions which are not unique. (Shewell, 1959) This ambiguity may be regarded as symptomatic of a defect in the quantum formalism. It is herein shown that the procedure employed in this work of expanding classical expressions in terms of de Broglie Fourier wave components, yield unique operator equivalents. Consider the following example:

$$< xp >= \int\int dx dp\, xp\, \rho(x, p). \tag{62}$$

With Eqs. (54) and (56), and retaining the order of x and p, this is

$$< xp >= \int\int\int dp\, p\, dr dr' \left(\frac{r + r'}{2}\right) e^{\frac{ip\cdot(r-r')}{\hbar}} \Psi * (r')\Psi(r), \tag{63}$$

which, after integration by parts, yields:

$$< xp >= \int dr \Psi^*(r) \left(r + \frac{\hbar}{i}\nabla\right) \Psi(r) + \frac{\hbar}{2}. \tag{64}$$

Similar manipulations with expressions of the form $< x^m p^n >$ also give unambiguous results, in contrast to the orthodox quantization rules. In can also be verified that the usual *ad hoc* procedure for building 'Hermitian' operator, for example for the Lorentz force law, (Schiff, 1968, p. 178), is here an automatic consequence of the procedure.

3. Decomposition: Resolution of Interpretational Issues

The inverse of this procedure was developed by Wigner in 1932, when he introduced what is nowadays called the Wigner distribution function:

$$\rho_w(x, p) = \int dx' e^{\frac{i2p\cdot x'}{\hbar}} \Psi^*(x + x')\Psi(x - x'), \tag{65}$$

and extracted from the Schrödinger equation an equation which is the Liouville equation with spurious terms:

$$\frac{\partial \rho}{\partial t} = -\mathbf{p} \cdot \frac{\nabla \rho}{m} + \sum_{j,k} \frac{(\hbar/2i)^{j-1}}{j!} \frac{\partial^i V(x)}{(\partial x^k)^j} \frac{\partial^i \rho}{(\partial p_k)^j}; \quad j = 1, 2, \ldots \tag{66}$$

The question now is, why has Eq. (65), which is no more than the inverse of the transformations Eq. (54) and (56), not resulted in Liouville's equation without the extra terms? The answer lies in the structure of Eq. (64), which would require the functions $\Psi(r)$ to be rigorously correct even off the line defined by $r = r'$, where (now) we know, in fact, that they do not satisfy Eq. (57). Note, however, that when V(x) is of the form:

$$V(x) = a + bx + cx^2, \tag{67}$$

Eq. (57) is separable and the solutions $\Psi(r)$ to Eq. (61) are rigorously valid over the entire $r - r'$ plane; in those circumstances a classically meaningful phase space distribution (Wigner density function) can be computed. As Wigner has pointed out, the discrepancy in the general case offers the possibility, in principle, of distinguishing between the orthodox

and any other interpretation more directly related to probability theory.

Wigner also has shown, that Eq. (65) leads to densities on phase space which are not everywhere positive when the function $\Psi(r)$ is an eigenfunction of the Schrödinger equation, Eq. (61); for example, when $\Psi(x)$ is the so-called first excited state of the harmonic oscillator:

$$\rho(x,p) = \int e^{-x^2+i2\mathbf{p}\cdot\mathbf{x}'-x'^2}(x^2-x'^2)dx', \quad (68)$$

$$= \sqrt{\pi}e^{-x^2}(x^2-1/2). \quad (69)$$

The observation that $\rho(x,p)$ is not positive definite was thought (by Wigner) to preclude the possibility of interpreting $\rho_w(x,p)$ as a density or probability. However, this argument implicitly assumes that eigenfunctions of Eq. (61), or energy eigenstates, have objective reality; that is, that a physics system can 'exist' in a pure energy eigenstate as opposed to mixture of eigenstates. This implicit assumption is clearly not germane to the present interpretation, as only those states, mixed or pure, which do give acceptable Wigner densities are regarded as physically meaningful; in other words: satisfy the initial and boundary conditions. Others, e.g., Marshall (1963) and Landé (1965a), have also come to the same conclusion. It is interesting that thermal states (i.e., mixed states with Boltzmann weighting factors) and coherent states satisfy this criterion. (Meijer, 1966)

Appendix to Subsection D

Claim: If $\Psi(r)$ satisfies:

$$\{\frac{\partial}{\partial t}+\frac{\hbar}{i2m}\nabla^2-\frac{i}{\hbar}rF(r)+C(r)\}\Psi(r)\equiv\{O\}\Psi(r)=0, \quad (70)$$

and $\Psi^*(r)$ its complex conjugate, then

$$\hat{\rho}(r,r')=\Psi^*(r')\Psi(r), \quad (71)$$

satisfies, when $r=r'$:

$$\{\frac{\partial}{\partial t}+\frac{\hbar}{i2m}\left(\nabla^2-\nabla'^2\right)-\frac{i}{\hbar}(r-r')F(\frac{r+r'}{2})\}\hat{\rho}(r,r')=0. \quad (72)$$

Proof: Put Eq. (71) into (72), add zero in the form $(C-C)\Psi^*\Psi$, rearrange to get:

$$\Psi^*\{O\}\Psi+\Psi\{O'\}\Psi*, \quad (73)$$

let $r\to r'$, and the conclusion follows.

Claim: The computation of expectation values of $<p^n>$ require the solution to Eq. (51) only on the line $r=r'$.

Proof:

$$<p>=\int\int dpdx\,p\,\rho(x,p), \quad (74)$$

Which with Eq. (48) and (50), becomes:

$$<p>=\int\int\int dxdx'dp\,p\,e^{\frac{-2p\cdot x'}{\hbar}}\hat{\rho}(x,x'), \quad (75)$$

and with Liebnitz's rule, becomes:

$$<p>=\int\int drdr'\,\delta(r-r')\left(\frac{\hbar}{-2i}\right)\left(\frac{\partial}{\partial r}-\frac{\partial}{\partial r'}\right)\hat{\rho}(r,r'). \quad (76)$$

The appearance of the Dirac delta function, proves the point.

Claim: $\rho(r)$, the configuration space density computed from $\rho(x,p)$ is uniquely determined by $\Psi(r)$ on the $r=r'$.

Proof: By definition

$$\rho(x)=\int dp\rho(x,p), \quad (77)$$

which with Eq. (51), becomes:

$$\rho(x)=\int\int dpdx'e^{\frac{-ipx'}{\hbar}}\rho(x,x'), \quad (78)$$

changing variables with Eqs. (53)

$$\rho(r)=\frac{1}{2}\int dr\delta(r-r')\Psi^*(r')\Psi(r) - \frac{1}{2}dr'\int dr'\delta(r'-r)\Psi^*(r')\Psi(r), \quad (79)$$

which is

$$\rho(r)=\Psi^*(r)\Psi(r), \quad (80)$$

upon accounting for the relative orientation of the $r=0$, $r'=0$ and $x=0$ lines and integrations.

II. RELATIVISTIC MECHANICS

In Section I, Quantum Theory was studied and a quantization procedure found which does not ascribe a particlar status to 'time' as an independent variable in the theory. This procedure consists of decomposing Liouville's equation in terms of Fourier components, each of which is a de Broglie wave, and then to find a solution on a restricted domain. In this Section a formulation for the application of this procedure will be presented. However, before discussing relativistic mechanics in Subsection B, theorems widely thought to be fatal regarding the formulation of relativistic canonical mechanics will be analyzed.

A. No-interaction Theorems

1. The Role of 'Time' in No-interaction Theorems

Here the first point is to show that no-interaction theorems all result from efforts to give 'time' the same special status

that it has in non-relativistic quantum mechanics. These theorems have received much attention, as they are thought to completely preclude the possibility of formulating of a canonical, i.e., Hamiltonian, formulation of relativistic mechanics with interaction. (Leutwyler, 1965) Thus, although a logical refutation would be achieved by displaying a counterexample, these theorems are analyzed here deeper to reveal their true bearing on the issue and to forestall misapprehensions regarding their impact.

There are two distinct types of no-interaction theorems. The first type was proffered by Currie, Jordan and Sudarshan (1963) (hereafter CJS), which can be characterized by the invariance requirement in its hypothesis, The second type, proposed by Droz-Vincent (1972), is characterized by the assumption that the mechanics of an N-particle system is described by an N-parameter group structure. The tactic here will be to show how the results of these theorems is obtained adding stipulations to mechanics as formulated by Cartan (1922); see also: Abraham (1967) and Slebodzinski (1970), for notation and vocabulary.

Let M_N be the direct product of the Minkowski spaces of each of N particles of a system. Let ω_N be the differential action-form of the system:

$$\omega_N = \sum_{n=1}^{N} \mathbf{p}^{\mathbf{n}}_{\mu} d\mathbf{x}^{\mu}_{\mathbf{n}} - \mathcal{H} d\tau, \tag{81}$$

where $\mathbf{x}^{\mu}_{\mathbf{n}}$ is the canonical 4-position of the n-th particle, $\mathbf{p}^{\mathbf{n}}_{\mu}$ its canonical momentum, $\mathcal{H}$ the total energy of the system (the Hamiltonian) and τ the variable conjugate to the Hamiltonian, and therefore the parameter of the generator for the dynamics. Now define a vector field, D, on the $8N+1$ dimensional space $M_N + 1$ as follows:

$$D \equiv V^1_1 \frac{\partial}{\partial x^1_1} + \ldots + V^4_N \frac{\partial}{\partial x^N_4} + F^1_1 \frac{\partial}{\partial p^1_1} + \ldots + F^1_N \frac{\partial}{\partial p^N_4} + \frac{\partial}{\partial \tau}. \tag{82}$$

Then, Cartan's principle requires that the interior product of the vector field D, with the exterior derivative of the action form ω_N, be zero, to wit:

$$i(D) dw_N = 0. \tag{83}$$

Eq. (83) then specifies the functions V^{μ}_n and F^n_{μ} such that they will satisfy Hamilton's Principle and such that the integral curves of D projected on M_N are the world lines of the particles. Further recall that

$$i(V_A) d(\omega_{M_N}) = dA, \tag{84}$$

where

$$\omega_{M_N} = \sum \mathbf{p}^{\mathbf{n}}_{\mu} d\mathbf{x}^{\mu}_{\mathbf{n}}, \tag{85}$$

defines an isomorphism between the vector fields V_A and the functions A. Also recall that, the Minkowski structure of each space-time manifold M_n requires that the basis elements of the tangent space to each M_n, when viewed as generators of translations, satisfy the Poincaré group relationships together with Lorentz transformations with respect to the Lie bracket.

2. The Special Role of Time in CJS Theorems

In the context of Cartan's formulation of mechanics, a CJS theorem's hypothesis contains the following stipulations: a) the canonical momenta are to be identified with generators of space translations and therefore satisfy the Poincaré group relations, and b) the Hamiltonian specifies the dynamics via the bracket relations

$$[\mathbf{x_n}, V_H] = V_n; \quad [\mathbf{p}^n, V_H] = F_n, \tag{86}$$

where $\mathbf{x}_n$ and $\mathbf{p}^n$ are canonical variables and V_H is obtained with Eq. (84), and finally, *in distinction from Cartan*, c) that V_H can be identified with the generators of time translations; i.e., that

$$\frac{\partial}{\partial \tau} = V_H = a^1 \frac{\partial}{\partial x^4_1} = \ldots = a^N \frac{\partial}{\partial x^4_N}. \tag{87}$$

Now, by putting Eq. (87) into Eq. (82) and applying Eq. (84), one finds

$$F^{\mu}_n = 0; \quad V^{\mu}_n = -a^n. \tag{88}$$

Obviously, as condition "c" is unique to a CJS type theorem, it is the source of the no-interaction result.

From the extensive commentary in presentations of CJS theorems, it can be inferred that the justification for condition 'c' is the need to have the dynamics specified by a canonical transformation. However, while a sufficient condition for the generator of the dynamics V_H to be canonical would be for it to be contained in the Poincaré group, it is not a necessary condition. It is necessary only that the generator preserve the symplectic structure; i.e., preserve the form of ω_N. Thus, this motivation for condition 'c,' must be incorrect. In fact a deeper motive for conditions 'c' is the hope of construction a variation of relativistic mechanics which would be amenable to the classical quantization procedures. Since these procedures, in fact all of relativistic quantum mechanics as presently formulated, give time a status different from that of space, relativistic mechanics defies direct quantization. Therefore, various authors have tried to reformulate mechanics using Lie groups whose brackets can be "translated" into operator commutators with the appropriate insertions of $i\hbar$, and so forth. However, when the Lie brackets are required to have the Poincaré group structure constants to make the theory relativistic, and when the Hamiltonian vector field V_H is identified with the time displacement operator to give time the role is has in quantum theory, a CJS theorem follows.

Recently, it has been shown that the conclusion of a CJS no-interaction theorem can be escaped by relinquishing the requirement that the position variables be canonical. (Kerner, 1965) This is tantamount to not requiring that the Poisson brackets of these variables be equal to zero; i.e.,

$$[x^i, x^j] \neq 0, \tag{89}$$

while retaining it for the momentum variables:

$$[p^i, p^j] = 0. \tag{90}$$

This has the undesirable consequence of entailing a complicated non-Minkowski metric on physical space, while retaining a Minkowski metric on momentum space, the difference being made up by the vector potential. Of course, one could go the other way around, retaining a Minkowski metric on physical space but no momentum space; in either case the resulting bracket relations defy quantization. Thus, one is lead into a cul-de-sac; a relativistic mechanics involving quantizable canonical variables cannot be formulated without treating time and space equivalently; and on the other hand, a single-time formulation of mechanics cannot accommodate quantizable canonical variables.

3. The Droz-Vincent No-interaction theorem: An exploration

The Droz-Vincent (DV) no-interaction theorem is essentially an exploratory study in response to the challenge posed by CJS theorems; however, it too yields the no-go result. The fundamental assumption of a DV theorem is that the dynamics are to be described by an N-parameter group structure. In the fashion of Cartan, this is tantamount to the definition of the vector field D, Eq. (82) as:

$$D = V_1^1 \frac{\partial}{\partial \mathbf{x}_1^1} + \ldots + F_4^N \frac{\partial}{\partial \mathbf{p}_4^N} + \frac{\partial}{\partial \tau^1} + \ldots + \frac{\partial}{\partial \tau^N}, \tag{91}$$

and ω_N as:

$$\omega_N = \sum p_\mu^n dx_n^\mu - \mathcal{H} d\tau^1 - \ldots - \mathcal{H} d\tau^N. \tag{92}$$

After some mildly tedious calculation following from application of Eq. (83), it follows that

$$\{\mathcal{H}^i, \mathcal{H}^j\} \equiv 0,$$

where the brackets are those of Poisson. The independence of the world line follows from the independence of the Hamiltonians.

In short, these theorems do not preclude the existence of a canonical formulations of relativistic mechanics. What they do preclude is a canonical formulation which exalts time to a status different from than of space. If canonical formulations with interaction of mechanics exist, they must remain true to the spirit of special relativity by keeping time and space in equivalent roles.

B. Relativistic Action-at-a-distance Mechanics

Field theories suffer the following inadequacy which might lead one to consider an action-at-a-distance (AD) formulation as being more fundamental. To quote Rohrlich (1965), "... a consistent action integral for an electromagnetic n-particle system ($n > 1$) requires the introduction of direct interaction between charges, not mediated by a field." In additions, AD mechanics avoids altogether the self-energy divergencies which plague field theories. However, the present form of relativistic AD mechanics, due essentially to Fokker (1929),[3] leads to mathematically "poorly posed," (i.e., non integrable) equations of motion, because it incorporates advanced as well as retarded interaction between particles. Wheeler and Feynman (1945/49), have argued that this repulsive feature, tantamount to predeterminism, does not violate our phenomenological understanding of causality. They argue that the inclusion of the whole universe somehow cancels the advanced part to within observable limits; however, the equations remain non integrable. It is the point of this Subsection to propose a change in Fokker's formulation of relativistic AD mechanics so that it does lead to well-posed equations of motion without sacrificing any of the aesthetic or mathematical criteria which Fokker demanded of a fundamental principle.

1. Fokker's Unified Lagrangian

Fokker, the principle originator of covariant mechanics, sought to generalize Newtonian mechanics to be consistent with a finite propagation speed of interaction by seeking a Lorentz invariant action integral for which variation would lead to n coupled equations of motion for an n particle system. He also demanded. largely on aesthetic grounds, that a fundamental action principle for interacting particle require the variation of a single integral, rather than a separate integral for each particle in the system. These criteria are met by the following Lagrangian proposed by Fokker:

$$\begin{aligned} \mathcal{L} = \sum_i^N \mathcal{L}_i = \sum_i^N & (-m_i c(\mathbf{x_i} \cdot \mathbf{x_i})^{1/2} \\ & - e_i \sum_{j \neq i} e_j \int_{-\infty}^{+\infty} \mathbf{x_i} \cdot \mathbf{x_j} \delta\left((\mathbf{x}_i(\lambda_i) - \mathbf{x}_j(\lambda_j))^2\right) d\lambda_j. \end{aligned} \tag{93}$$

as a result of the variation:

$$\delta \int \sum \mathcal{L}_i \, d\lambda_i = 0. \tag{94}$$

Here, all vectors $\mathbf{x}$, are Minkowski four-vectors, $\dot{\mathbf{x}} = d\mathbf{x}/d\lambda_i$, where λ_i is each particle's proper-time, and $\delta\left((\mathbf{x}_i(\lambda_i) - \mathbf{x}_j(\lambda_j))^2\right)$ is the Dirac delta function (all conventions here are in accord with (Hanus and Janyszek, 1971), except the explicit form of Eq. (93)).

A careful inspection of Eq. (93) reveals that two contributions occur for each time the following is true:

$$|\mathbf{x_i}(\lambda_i) - \mathbf{x_j}(\lambda_j)| = 0. \tag{95}$$

In other words, each interaction is made to serve double duty, once as a retarded interaction, i on j say, and once as an advanced interaction j on i. In this way Fokker was able to combine the separate variational integrals of each particle to obtain a single integral for the system. By way of contrast, the

[3] Subsequent study revealed that Karl Schwartzschild and Walter Ritz are more deserving of credit as the originators. Fokker supported advanced interaction, however.

Lagrangian

$$\mathcal{L} = \sum_i^N \mathcal{L}_i = \sum_i^N (-m_i c(\mathbf{x_i} \cdot \mathbf{x_i})^{1/2} - e_i \sum_{j \neq i} e_j \int_{-\infty}^{\lambda_{ij}} \mathbf{x_i} \cdot \mathbf{x_j} \delta\left((\mathbf{x}_i(\lambda_i) - \mathbf{x}_j(\lambda_j))^2\right) d\lambda_j., \quad (96)$$

where λ_{ij} is that value of λ_j which includes only the retarded interaction, does not lead to advanced interaction. However, Eq. (96) fails to be a *single* integral because each interaction must be written twice, once with λ_{ij}, and once with λ_{ji}, and may, therefore, be written as a separate integral for each particle. Thus, Fokker found himself choosing between a single integral with advance interaction, and separate integrals with advanced interaction.

2. A Unified Lagrangian without Mathematical Lacunae

Escape from this cul-de-sac can be had with following single parameter Lagrangian

$$\mathcal{L} = \sum_i^N \mathcal{L}_i = \sum_i^N (-m_i c(\mathbf{x_i} \cdot \mathbf{x_i})^{1/2} - 2e_i \sum_{j \neq i} e_j \int_{-\infty}^{\tau} \mathbf{x_i} \cdot \mathbf{x_j} \delta\left((\mathbf{x}_i(\tau) - \mathbf{x}_j(\tau'))^2\right) d\tau'. \quad (97)$$

The variation

$$\delta \int \mathcal{L} d\tau = 0, \quad (98)$$

then, yields n coupled equations of motion

$$m_i \ddot{\mathbf{x}}_\mathbf{i}^\mu = \frac{e_i}{c} \sum_{j \neq i} (\mathbf{F}_{\text{ret.}})_j^{\mu\nu} (\dot{\mathbf{x}}_i(\tau))_\nu. \quad i = 1, \ldots, N, \quad (99)$$

if the scale of τ is adjusted to satisfy

$$\dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i = c^2; \quad i = 1, \ldots, N, \quad (100)$$

This condition is necessary in order to obtain Eq. (99) in a simpler form, and to completely conform with Special Relativity by forcing velocity vectors to be normalized to c.[4]

Eq. (99) differs from those derived from Fokker's Lagrangian, namely Eq. (93) and those from the Lagrangian, Eq. (96)

$$m_i \ddot{\mathbf{x}}_\mathbf{i}^\mu(\lambda_i) = \frac{e_i}{c} \sum_{j \neq i} (\mathbf{F}_{\text{ret.}})_j^{\mu\nu} (\dot{\mathbf{x}}_i(\lambda_i)_\nu. \quad i = 1, \ldots, N, \quad (101)$$

in that Eqs. (99) have a single independent variable τ, and involve only retarded interaction. Furthermore they enjoy the quality of meeting Fokker's aesthetic criterion of being derived from a single variational principle. In fact, the unification achieved by Lagrangian Eq. (97) is of a deeper sort than achieved by Fokker's Lagrangian in that it results from the use of a single independent parameter, rather than the technicality of being unable to write the Lagrangian as the sum of single Lagrangians. Eqs. (99) also enjoy the property of being mathematically well-posed since they are coupled one to another by retarded interaction in a single parameter structure of an intrinsic, rather than *ad hoc* nature.

Wheeler and Feynman (1945/49) have given extensive philosophical apologetics for the inclusion of advanced interaction; however, Eqs. (93) are nevertheless poorly-posed in the sense that solutions cannot be found by local integration, even if they exist. This becomes obvious if one imagines a machine integration of the i-th equation for a given value of λ_i. Such an integration; i.e., a calculation of an extention of the orbit for an incremental increase in λ_i, requires information from the j-th orbit on the forward light-cone of the i-th particle, but this portion of the orbit is yet to computed, etc., *ad infinitum*. In effect, one needs the solutions to the equations as initial data in order to compute solutions! Although the equations resulting from Lagrangian Eq. (96) can be integrated, that can also be faulted since each equation has its own parameter λ_i. In a machine calculation, λ_i would be an incrementation parameter, and care would have to be taken to insure that all such incrementation parameters advanced such that the separate equations remain compatible. Clearly a single parameter formulation, would be more efficient in this respect. Moreover, the sort of equations resulting from Eq. (97) have been studied in another context by Driver (1963), who has shown that in the one dimensional case unique solutions exist given a compact set of initial data.

The role of the single parameter τ, in the light of the history of Special Relativity, is somewhat unclear. It can be taken that τ has no physical significance, but is simply an 'unraveling' parameter of the world lines of the system particles; i.e., its role is purely mathematical. The geometric structure of a single parameter formulation of mechanics, irrespective of the nature of the metric, has been studied by Cartan, who clearly shows that τ is the independent parameter in the generator of the dynamical group on the $4 \times n$ dimensional configuration manifold. It is a fact of no small import for the foundations of mechanics, that only a single parameter Lagrangian is compatible with Cartan's Principle, in that Cartan's analysis is rigorous *global* mathematics. Thus, τ has indisputable mathematical importance, whatever physical sense with regard to the experience of time it might also be given.

In short, two objectives have been achieved by these considerations, one, interpretational, one, strictly mathematical. The first is a demonstration that a formalism of relativistic action-at-a-distance mechanics not incorporating advanced interaction is possible. Additionally, it has been shown here that there is a formulation free of the mathematical defect of yielding poorly-posed equations of motion, making it a consistent and natural extention of Newtonian mechanics suitable for quantization.

[4] *After-the-fact note:* This condition, as such, is unnecessary as it is automatically fulfilled by definition.

C. Interpretational and Self-consistency Issues

There are, at least, two obstacles to the more-or-less explicit claim made in the foregoing discussion that an AD formulation of electrodynamic interactions is more 'fundamental' that field theoretic formulations. "Fundamental" here is used in the philosophical rather than practical sense; that is: it is not meant to imply that an AD formulation is more convenient, or even feasible, for doing calculations, or understanding applications of electrodynamics better, ore even at all, in such terms. It is meant only that the basic nature of interaction between point charges is best revealed in this formulation and that questions on its character that make no sense in an AD formulation, in fact make no sense absolutely. An example of such a question is: how much energy is contained in the field of a point charge? As this question is meaningless in AD, it can be taken as absolutely meaningless altogether.

The obstacles mention above are: the hypothetical existence of magnetic monopoles, and the nature of radiation reaction.

1. Magnetic Monopoles: an Inherent Contradiction

The existence of magnetic monopoles would be a serious obstacle for accepting the validity of AD mechanics as has been shown by Havas (1957) and Carter and Cohen (1973). There existence in the first place was proposed on the basis of the symmetry they would bring to the Maxwellian, field theoretic formulation of electromagnetic interaction by Dirac (1948). However, enthusiasm has lead to a cavalier resolution of the following inconsistency among the formula pertaining to such monopoles.

Consider the equation of motion for a pure magnetic monopole of strength q, mass m in the presence of fields $\mathbf{E}$ and $\mathbf{B}$:

$$m\ddot{\mathbf{x}} = q(\mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E}). \tag{102}$$

In order for this equation to be mathematically consistent, either q or m must be a pseudoscalar, because both $\mathbf{B}$ and $\mathbf{v} \times \mathbf{E}$ are pseudovectors. Both choices lead to conflict.

If m is taken as a pseudoscalar, then q would have to be a scalar, which leads to a vector (vise pseudovector) $\mathbf{B}$ field. This in turn would lead to an inconsistent equation of motion for an electric charge exposed to this field:

$$m_e\ddot{\mathbf{x}} = e(\frac{\mathbf{v}}{\mathbf{c}} \times \mathbf{B_q}). \tag{103}$$

On the other hand, if q is chosen to be a pseudoscalar, then Dirac's quantization condition:

$$qe = \frac{n\hbar c}{2}, \tag{104}$$

is inconsistent, because all the other factors are scalars! This problem is skirted in the literature, with the folkloric suggestion that Eq. (104) be interpreted to be:

$$|q|e = \frac{n\hbar c}{2}; \tag{105}$$

there is, however, no consistent argument supporting such a modification.

The failure to find magnetic monopoles to date, can be taken as support for the rectitude of an AD formulation vís-a-vís field theory; perhaps, even, as a symptom of a fundamental weakness of field theories altogether.

2. Radiation Reaction: An Emergent, Multi-body Effect

In (Dirac, 1938) the expression for the force due to radiation reaction, and in the AD formulation developed by Wheeler and Feynman (1945/49), advance interaction plays a vital role. In the Wheeler-Feynman formulation, the electromagnetic force on a particle results in part from the advanced signal of a hypothetical 'adsorber at infinity' responding to the retarded field of the charge under consideration. Although eliminating advanced interaction only cleans up the latter story, advanced interaction for radiation reaction must be rationalized.

The paradigm proposed here to explicate radiation reaction has certain similarities with the Wheeler-Feynman formulation in that it employs the reaction of the rest of the universe, which is taken to be on the whole electrically neutral. In such a universe, a particle in an AD formulation is considered to be in permanent interaction, at least via electrostatic forces, with all other charges in the universe, and would, therefore, statistically seen, induce a Debye sheath around itself. To first approximation, this sheath can be considered as a coincident "hole" of opposite charge to the particle, with which the interaction constitutes 'radiation reaction.' The coupled equations for a charge would then be:

$$m(\ddot{x}_p)^\mu = \frac{e}{c}(F_h)^{\mu\nu}_{\text{ret.}}(\dot{x}_p)_{\nu,} \tag{106a}$$

and for its hole

$$m(\ddot{x}_h)^\mu = -\frac{e}{c}(F_p)^{\mu\nu}_{\text{ret.}}(\dot{x}_h)_{\nu.} \tag{106b}$$

The solution of this system is facilitated by the following approximations: To first order $\mathbf{x_p} = \mathbf{x_h}$ (up to effects of time lag to be discussed below). Then, the retarded field from the hole; i.e., from other charges in the universe organized as an 'onion' like Debye sheath around the particle, will appear to be a charge-reversed, converging field mimicking a time-reversed diverging field; that is: the imploding field of the sheath is identical to time-reversed and sign-changed particle field, or purely formally (but in approximation):

$$(F_h)_{\text{ret.}} = (F_p)_{\text{adv.}}. \tag{107}$$

Therefore, Eqs. (106) can be added to give:

$$m(\ddot{x}_p)^\mu = \frac{e}{2c}((F_p)_{\text{ret.}} - (F_p)_{\text{adv.}})^{\mu\nu}(\dot{x}_p)_{\nu.} \tag{108}$$

This equation is precisely the starting point for Dirac's derivation of the expression for radiation reaction, for which this paradigm offers a new and intuitive understanding.

III. A RELATIVISTIC WAVE EQUATION WITH INTERACTION

A. An Equation for Bosons

In this Section, the quantization procedure used in Section I is applied to the AD mechanics developed in Section II, to give a manifestly covariant wave equation with interaction.

Mimicking the procedure in Section II on the single parameter Hamiltonian:

$$\frac{\partial \rho}{\partial \tau} = \frac{\partial \mathcal{H}}{\partial p}\frac{\partial \rho}{\partial x} - \frac{\partial \mathcal{H}}{\partial x}\frac{\partial \rho}{\partial p}, \tag{109}$$

yields:

$$\hbar\frac{\partial \Psi}{\partial \tau} = \sum_i^N \left(\frac{1}{2m_i}\right)\left(\hbar\Box_i - \sum_{j\neq i}^N A_j\right)^2 \Psi, \tag{110}$$

where

$$\Box_i = \nabla_i + \frac{\partial}{\partial(ict_i)}. \tag{111}$$

The appropriate Hamiltonian derived from the Lagrangian Eq. (97) is

$$\mathcal{H} = \sum_i^N \frac{1}{m_i}\left(\mathbf{p}_i - \sum_{j\neq i}^N A_j\right)^2, \tag{112}$$

where

$$A_j = e_j \int_{-\infty}^{\tau} \mathbf{x}_j(\tau')\delta\left((\mathbf{x}_i(\tau) - \mathbf{x}_j(\tau'))^2\right) d\tau'. \tag{113}$$

Eq. (110) is manifestly covariant, all vectors are expressed in four-vector form, and the interactions terms are non-null only on the light cone.

The use of the Liouville equation in a relativistic setting automatically implies a change in the calculations role of $\Psi^*\Psi$, in that the density $\rho(\mathbf{x},\mathbf{p})$ on a space-time manifold rather than configuration space as in non relativistic mechanics. Thus, this density cannot be interpreted a probability of presence or density of world lines, rather as probability of a space-time measurement or 'event.' The probability of presence is then, as is usual in relativistic quantum theory, the fourth component of the current computed with $\Psi^*\Psi$:

$$\mathbf{j} = \frac{1}{m}\Psi^*\mathbf{p}\Psi.$$

As a candidate for a relativistic multi-body wave equation, Eq. (110) in not entirely without precedent in the physics literature. Feynman (1951) expressed a preference for a second order equation over a first order Dirac equation, because it is more amenable to analysis using Green's functions; and, he also investigated the introduction of a fifth parameter (Feynman, 1958). Arunasalam (1970) also has proposed the single particle version and has studied many of the consequences of the format. An AD formulation of Relativistic quantum theory, however, brings with it issues that remain to be studied, for example the exclusion of negative energies, (Hanus and Janyszek, 1971), and the definitions of space-time operators; see, e.g., Sakurai (1967). The more serious challenge, however, will be to find a suitable application for which the equations are solvable.

B. A Fermion Equation

Pauli (1927), apparently, was first to publish the suggestion that altering the Schrödinger equation, even though nonrelativistic, to read:

$$\hbar\dot{\Psi} = \frac{1}{2m}\left(\sigma\cdot(\hbar\nabla - \frac{e}{c}A)\right)^2\Psi + \Phi\Psi, \tag{114}$$

leads to the correct magnetic moment for the electron (modulo anomalies). Later it was seen by many authors that the same alteration to the Klein-Gordon equation gives a second order equation equivalent to the Dirac equation. (Sakurai, 1967) Thus, the same alteration should render Eq. (110) suitable for the description of Fermions. Simply following this example, however, violates the spirit of this study to provide intuitive motivation; so that, herein, it shall be argued that this formal procedure is a means to take the two polarization states of the background into full account.

By introducing the Pauli spin matrices, the equations of motion can be coupled in a way compatible with the coupling of the two modes of polarization of light—a fact which accords fully with the fact that spin operators were in use as "Stokes" polarization operators decades before spin was introduced. Thus, as a tandem Liouville equation, consider:

$$\begin{aligned}\dot{\rho}_1 &= (\mathbf{p}\cdot\nabla - \mathbf{F}\cdot\nabla_p)\rho_1;\\ \dot{\rho}_2 &= (\mathbf{p}\cdot\nabla - \mathbf{F}\cdot\nabla_p)\rho_2,\end{aligned} \tag{115}$$

where ρ_1 is the density of orbits in equilibrium with one state of polarization, and ρ_2 the other. Now, by virtue of the identity:

$$(\sigma\cdot\mathbf{A},\Pi A_4)(\sigma\cdot\mathbf{B},\Pi B_4) = \Pi\mathbf{A}\cdot\mathbf{B} + i\sigma\cdot(\mathbf{A}\times\mathbf{B}) - \Pi A_4 B_4, \tag{116}$$

these equations can be rewritten as

$$\left\{-\frac{d}{d\tau} + \mathcal{S}\right\}\begin{bmatrix}\rho_1 & 0\\ 0 & \rho_2\end{bmatrix} = 0, \tag{117}$$

where

$$\mathcal{S} = (\sigma\cdot\mathbf{p})(\sigma\cdot\nabla) - (\sigma\cdot\mathbf{F})(\sigma\cdot\nabla_p) - i\sigma\cdot(\mathbf{p}\times\nabla - \mathbf{F}\times\nabla_p). \tag{118}$$

Continuing as earlier, use as a Fourier kernel a solution of a pair of wave equations:

$$\mathcal{K} = exp(i(\sigma\cdot\mathbf{k})(\sigma\cdot\mathbf{x}) + \Pi\omega t, \tag{119}$$

which it is easy to see is the solution of a 'paired' wave equation:

$$\left\{(\sigma\cdot\nabla)^2 - \frac{\Pi}{c^2}\frac{\partial^2}{\partial t^2}\right\}\mathcal{K} = 0. \tag{120}$$

So far this is just play with algebraic structure to be use for convenience; it is not intended to be physically or mathematically meaningful beyond the obvious.

Eq. (45) permits replacing $\sigma \cdot \mathbf{k}, \Pi\omega$ with

$$[\frac{1}{\hbar}2\sigma \cdot \mathbf{p},\ \frac{1}{\hbar}2\Pi E]$$

in Eq. (119), so that it can be used as a "tandem" Fourier" kernel on the "tandem" Eqs. (117) as follows:

$$\int d(\sigma \cdot \mathbf{p}), \Pi E) e^{\frac{i2\sigma \cdot \mathbf{p}}{\hbar}, \frac{\Pi E}{\hbar}} \times \mathrm{Equ}(120). \qquad (121)$$

The result is similar to Eq. (56), again with an arbitrary function C, which is to be chosen so that the resulting equations will satisfy a "tandem" Ehrenfest type proof lead to a "tandem" generalization of Newton's Second Law:

$$\left(\frac{d^2}{d\tau^2}\right) \begin{bmatrix} <\mathbf{x}>_1 & 0 \\ 0 & <\mathbf{x}>_2 \end{bmatrix}$$
$$= -\begin{bmatrix} <\partial\mathcal{H}/\partial\mathbf{x}>_1 & 0 \\ 0 & <\partial\mathcal{H}/\partial\mathbf{x}>_2 \end{bmatrix} + <\nabla\frac{e\hbar}{2mc}\sigma \cdot \mathbf{B}> . \qquad (122)$$

The expectation $< \cdots >_i$ is computed with ρ_i and represents a density of 'events' on orbits of particles deemed to be in equilibrium with background waves of the i-th polarization mode. The result of all the above is the following equation:

$$i\hbar\frac{d\Psi}{dt} = \sum_i^N \frac{1}{2m_i}\left(\sigma \cdot \nabla_i + \Pi\frac{\partial}{\partial ict} - \sigma \cdot A_i - \Pi A_4\right)^2 \Psi, \quad (123)$$

where the square is to be executed using Eq. (116). The clumsy avoidance of Dirac matrices here was intended to avoid introducing negative energy, pending a suitable interpretation in the context of AD mechanics.

C. Conclusions

All in all, solutions for four important theoretical physics problems have been proposed herein. The merit theses solutions have is a question for the long run. No one is compelled to to accept them; but, for problems which have long resisted attack, virtually any proposal should be welcome. Ultimately, their merit will be determined by their contribution to sensible and verifiable calculations, or the prediction of heretofore unknown phenomena. On the other hand, they might also contribute to a better interpretation or philosophical understanding of fundamental physics.

The following considerations are preliminary judgments against these goals.

1. Duality and Wave-packet Collapse are Eliminated

The understanding of viewpoint on duality afforded by the imagery proposed herein offers a solution to what has been though of as two problems. First of all, the philosophical ambiguity surrounding the Copenhagen concept of 'duality' is swept away. In the process Einstein's viewpoint is vindicated.

> I am, in fact, firmly convinced that the essential statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems. (Schlipp, 1949, p. 666)

If the solution proposed herein is accepted, then it is the case that the wave function yields a phase-space density of Gibbsian ensembles. The Copenhagen argument that the wave function must have physical significance because it somehow mediates in physical interactions when beams pass through, e.g., slits or crystals, is seen to be unnecessary. The physical effect of the diffraction of beams is accomplished by the electromagnetism radiation in the background. Furthermore, the collapse of wave packets is seen to be non problematic because the density is a Gibbsian density. The philosophical impact of this viewpoint is immense. Most everything said based on quantum theory about uncertainty and determinism can be refuted. The following statements culled from Čapek (1961), characterize notions engendered by conventional quantum theory:

> ... the observed statistical laws of [microphysics] are not mere surface phenomena, ultimately reducible to classical causal models; on the contrary, the statistical laws are regarded as ultimate and irreducible features constituting the objective physical reality. ...radioactive explosions are regarded as *contingent events* whose irreducible chance character manifests the basic indeterminacy of microphysics occurrences.

The viewpoint introduced herein refutes these statements by exhibiting a conceptual scheme in which the "irreducible" statistical apparatus is divested of its preternatural behavior as an ethereal "statistical fluid," or whatever. In this viewpoint, the statistical apparatus is used to describe particles which are strongly influenced by background radiation. The first order effect of the background is to effectively attach a wave to the particle. It is the "attached wave" which causes the wavelike behavior described statistically with the machinery of a Gibbsian ensemble treatment.

A second feature of quantum theory which has led to an extreme number of philosophical speculations is the so-called "superposition principle." The fact that a statistical gadget does not follow the mathematical patterns of ordinary probability is the source of these speculations. However, the following rejoinders flow from the theory presented herein. One, the density used in quantum theory is unlike an ordinary density in that it can be manipulated to obtain statistical information regarding twice as many variables as are displayed as arguments of the density function. The burden of this additional structure is clearly revealed by the fact that the equation for the density itself, rather than the wave function, is nonlinear. Quite commonly, the sum of solutions to nonlinear equations is not a solution of the equation, and this fact can be found the peculiarities of the quantum superposition principle. It thus appears that these strictly mathematical facts divest the superposition principle of epistemological consequences.

However, the calculational merit of this viewpoint appears to be somewhat limited. It appears that the calculational merit may be found in the exception to the concept. Specifically, since the wave-like behavior is a first order "gross" response to the background, second order detailed effects may be amenable to calculations. An example of such a calculation is that of Welton (1948), for the Lamb shift. He assumes that the ground state of the quantized radiation field, identical in character to the classical electromagnetic background assumed herein, makes an electron wobble in orbit about the nucleus, which causes a perturbation to its energy to within a small fraction of the total shift. It seems a reasonable conjecture, that such a calculation might be made exact and extended to explain the anomalous magnetic moment of the electron. Interestingly, these second order effects are precisely those which it appears Einstein might have pointed to as not described by information contained in wave functions.

2. Single Parameter Relativistic Mechanics Precludes Predeterminism

The epistemological impact of predeterminism in Fokker's formulation of relativistic dynamics has not been large. It is not clear why this is so, but it may be supposed that the idea is so repulsive that philosophers have, for the most part, simply chosen to wait for its demise. Therfore, the philosophical impact of a reformulation not involving predeterminism is less.

The calculational significance of well formulated equations of motion is, on the other hand, potentially large. However, the equations are of sort not analytically well studied, and solutions of useful cases may very well turn out to be elusive.

The inherent symmetry of the concept of radiation reaction due to all other particles in the universe, vís-a-vís the concept of background radiation is philosophically charming. On the one side, radiation reactions is the mechanism which would cause an atom to collapse; on the other side the background is the source of energy holding an atom "up." In effect, different aspects of the same thing balance to yield a net stability.

3. Benchmark and Guidepost

The fact that a manifestly covariant equation with interaction can be written down is at least a guidepost. Where field theories have failed, AD has not. It is interesting to see that Eq. (123) can be separated into a τ independent equation whose eigenvalues are directly related to proper mass. In fact, if this is done for the Boson equation and the proper choice of eigenvalue made, Eq. (110) reduces to the Klein-Gordon equation. The same choice of mass gives the Dirac equation, except for negative energy, for the Fermion equation. This suggests that the eigen spectrum of Eq. (123) with an auspicious choice of interaction term might give the long sought mass spectrum for elementary particles. However, it is not clear if the interaction should be scalar, vector or whatever, or if another sort of background causes 'strong' and 'weak' interaction to be quantized at yet a lower level. These speculations are left for future study.

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